Magnetic line trapping and effective transport in stochastic magnetic fields

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The transport of collisional particles in stochastic magnetic fields is studied using the decorrelation trajectory method. The nonlinear effect of magnetic line trapping is considered together with particle collisions. The running diffusion coefficient is determined for arbitrary values of the statistical parameters of the stochastic magnetic field and of the collisional velocity. The effect of the magnetic line trapping is determined. New anomalous diffusion regimes are found.

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I. INTRODUCTION

The problem of test particle diffusion in stochastic magnetic fields has been studied by many authors [1–15] and important progress was obtained. However, the general solution has not yet been found. Particle trajectories in a magnetized plasma are determined by three stochastic processes: the magnetic field, the collisional velocity along magnetic lines, and the collisional velocity perpendicular to the magnetic lines. These components of the stochastic collisional velocities have very different effects. There are two important difficulties appearing in this triple stochastic process. One is related to the parallel collisional velocity which enters as a multiplicative noise in the equations of motion and the other with the Lagrangian nonlinearity which is determined by the space dependence of the stochastic magnetic field. Each of these two problems has been recently studied, but only considered separately. The complete model for particle transport in stochastic magnetic fields was not analyzed until now.

The problem is simplified whenever the study is restricted to stochastic magnetic fields with small amplitudes and/or large perpendicular correlation lengths, for which the magnetic Kubo number (defined below) is small. In this case the Lagrangian nonlinearity does not play an important role and the effect of the parallel collisional velocity could be determined. If the perpendicular collisional velocity is neglected, this "quasilinear" problem has an exact solution that was obtained by several methods [16]. It shows that the parallel collisional motion determines a subdiffusive transport across the confining magnetic field with the running diffusion coefficient $D(t)$ decaying to zero as $D(t)\sim t^{-1/2}$. It was shown [15] that this subdiffusive transport is due to collision-induced trajectory trapping along the magnetic lines. The parallel collisional velocity forces the particles to return along the magnetic lines in the correlated region and consequently generates the long time Lagrangian correlation of the stochastic velocity. If the perpendicular collisional velocity is taken into account, the transport is diffusive and the diffusion coefficient was estimated by several methods [1–14].

On the other hand, the Lagrangian nonlinearity determined by the space dependence of the stochastic magnetic field was studied in the related problem of particle diffusion in electrostatic turbulence. The equations of the $E\times B$ drift motion are mathematically identical with those for the magnetic line "evolution" along the direction of the main magnetic field. It was shown [17,18] that a process of trajectory trapping by the irregularities of the fluctuating electrostatic potential appears at large Kubo numbers and strongly influences the diffusion. Recently, a new statistical approach, the decorrelation trajectory method [19,20] was developed. It provides a rather detailed analytical analysis of the transport in the presence of trapping and evaluates the Lagrangian correlation and the running diffusion coefficient for arbitrary values of the Kubo number. Translated to the stochastic magnetic field case, this trapping process leads to localized segments of the magnetic lines with helicoidal shapes at large Kubo numbers.

The aim of this paper is to study the general problem of collisional particle diffusion in a stochastic magnetic field in the guiding center approximation. More specifically, we determine the effect of the magnetic line trapping on the effective transport. The running diffusion coefficient is determined for arbitrary parameters of the stochastic magnetic field and of particle collisions. The decorrelation trajectory method is used for studying this rather complicated triple stochastic process. We show that the magnetic line trapping can strongly modify the diffusion coefficient and determines anomalous diffusion regimes. The physical parameters corresponding to these regimes are determined.

The paper is organized as follows. The physical model is described in Sec. II. We derive in Sec. III the Lagrangian velocity correlation and the running diffusion coefficient for arbitrary values of the four specific parameters and for given Eulerian correlation of the potential. This general result is then analyzed by considering several particular cases of physical interest: the subdiffusive transport in Sec. IV, the effect of collisional cross-field diffusion in Sec. V and the effect of a time variation of the stochastic magnetic field in Sec. VI. The conclusions are summarized in Sec. VII.

II. THE SYSTEM OF EQUATIONS

The particle guiding center motion is studied in a magnetic field with a stochastic component. The magnetic field is taken to be a sum of a large constant field $\mathbf{B}_0 = B_0\mathbf{e}_z$ and a small fluctuating field perpendicular to $\mathbf{B}_0$, and depending on the perpendicular coordinates $\mathbf{x}\equiv(x,y)$ and on the parallel coordinate z,

$$\mathbf{B}=B_0(\mathbf{e}_z+\tilde{\mathbf{b}}(\mathbf{x},z,t)). \tag{1}$$

(Here the perpendicular and the parallel directions are defined in relation to the direction of $\mathbf{B}_0$.) This is the usual slab model of the confining configuration in a tokamak plasma. Since the reduced magnetic field is divergence-free, $\nabla\cdot\tilde{\mathbf{b}}=0$, its two components can be determined from a scalar function $\tilde{\phi}(\mathbf{x},z)$ as

$$\tilde{\mathbf{b}}(\mathbf{x},z,t)=\nabla\times\tilde{\phi}(\mathbf{x},z,t)\mathbf{e}_z. \tag{2}$$

The system of equations for guiding center motion (to dominant order) is

$$\frac{d\mathbf{x}}{dt}=\tilde{\mathbf{b}}(\mathbf{x},z,t)\,\eta_\parallel(t)+\boldsymbol{\eta}_\perp(t), \tag{3}$$

$$\frac{dz}{dt}=\eta_\parallel(t). \tag{4}$$

The three stochastic functions $\tilde{\mathbf{b}}(\mathbf{x},z,t)$, $\boldsymbol{\eta}_\perp(t)$, and $\eta_\parallel(t)$ are statistically independent: all cross correlations are zero. All these stochastic functions are assumed to be Gaussian, stationary, and homogeneous, with zero averages. The autocorrelation function of the stochastic potential $\tilde{\phi}(\mathbf{x},z,t)$ is modeled by

$$A(\mathbf{x},z,t)\equiv\langle\tilde{\phi}(\mathbf{0},0,0)\tilde{\phi}(\mathbf{x},z,t)\rangle =\beta^2\lambda_\perp^2\exp\left(-\frac{z^2}{2\lambda_\parallel^2}-\frac{x^2+y^2}{2\lambda_\perp^2}\right)\exp\left(-\frac{|t|}{\tau_c}\right), \tag{5}$$

where $\langle\cdots\rangle$ is the average over the realizations of the stochastic potential $\tilde{\phi}$, β is the mean square value of the reduced magnetic field $\tilde{\mathbf{b}}$, $\lambda_\parallel$ is the correlation length of the potential $\tilde{\phi}$ along the main magnetic field $\mathbf{B}_0$, $\lambda_\perp$ is the correlation length in the plane perpendicular to $\mathbf{B}_0$, and τ_c is the correlation time of $\tilde{\phi}$. The autocorrelation tensor of the reduced magnetic field components $B_{ij}\equiv\langle\tilde{b}_i(\mathbf{0},0,0)\tilde{b}_j(\mathbf{x},z,t)\rangle$, $i,j=x,y$, is determined from $A(\mathbf{x},z)$ as

$$B_{xx}=-\frac{\partial^2}{\partial y^2}A,\qquad B_{yy}=-\frac{\partial^2}{\partial x^2}A,\qquad B_{xy}=\frac{\partial^2}{\partial x\partial y}A. \tag{6}$$

The collisional velocities are modeled by colored noises with the correlations

$$\langle\eta_\parallel(0)\,\eta_\parallel(t)\rangle_c=\chi_\parallel\nu R(\nu t), \tag{7}$$

$$\langle\eta_\perp^i(0)\,\eta_\perp^j(t)\rangle_c=\delta_{ij}\chi_\perp\nu R(\nu t), \tag{8}$$

where $\langle\cdots\rangle_c$ is the average over the collisional velocity realizations, ν is the collision frequency, $\chi_\parallel=\lambda_{mfp}^2\nu/2$ is the parallel collisional diffusivity, λ_{mfp} is the parallel mean free path, $\chi_\perp=\rho_L^2\nu/2$ is the perpendicular collisional diffusivity and ρ_L is the Larmor radius relative to the reference field. $R(\nu t)$ is a time decreasing function that is chosen as

$$R(\nu t)=\exp(-\nu|t|) \tag{9}$$

for the explicit calculations presented in this paper.

We introduce dimensionless quantities in Eqs. (3) and (4) with the following units: $\lambda_\perp$ for the perpendicular displacements, $\lambda_\parallel$ for the displacements along the reference magnetic field and ν^{-1} for the time. The perpendicular velocity $\tilde{\mathbf{v}}=\tilde{\mathbf{b}}\eta_\parallel$ is reduced with $V\equiv\beta\sqrt{\chi_\parallel\nu}$, the parallel velocity $\eta_\parallel(t)$ with $\sqrt{\chi_\parallel\nu}$, and the perpendicular collisional velocity $\boldsymbol{\eta}_\perp(t)$ with $\sqrt{\chi_\perp\nu}$. The equations of motion in these dimensionless variables (denoted by the same symbols as the physical ones) are

$$\frac{d\mathbf{x}}{dt}=M\tilde{\mathbf{b}}(\mathbf{x},z,t)\,\eta_\parallel(t)+\overline{\chi_\perp}^{1/2}\boldsymbol{\eta}_\perp(t), \tag{10}$$

$$\frac{dz}{dt}=\overline{\chi_\parallel}^{1/2}\eta_\parallel(t). \tag{11}$$

Four dimensionless parameters appear naturally in this problem: the dimensionless perpendicular and parallel diffusivities, respectively,

$$\overline{\chi_\perp}\equiv\frac{\chi_\perp}{\lambda_\perp^2\nu},\qquad\overline{\chi_\parallel}\equiv\frac{\chi_\parallel}{\lambda_\parallel^2\nu}, \tag{12}$$

a dimensionless parameter that contains the effect of the stochastic magnetic field

$$M=\frac{V}{\lambda_\perp\nu}=\frac{\beta\lambda_\parallel}{\lambda_\perp}\overline{\chi_\parallel}^{1/2}, \tag{13}$$

and the dimensionless decorrelation time

$$\overline{\tau_c}=\tau_c\nu. \tag{14}$$

We note that the parameter which describes the evolution of the magnetic lines, the magnetic Kubo number $K_m=\beta\lambda_\parallel/\lambda_\perp$, appears here as a factor in M, which can be written as $M=K_m\overline{\chi_\parallel}^{1/2}$.

The aim of our calculation is to determine the Lagrangian autocorrelation of the effective perpendicular velocity

$$\tilde{\mathbf{v}}(\mathbf{x},z,t)\equiv\tilde{\mathbf{b}}(\mathbf{x},z,t)\,\eta_\parallel(t), \tag{15}$$

which leads to the effective perpendicular diffusion coefficient.

III. SOLUTION BY THE DECORRELATION TRAJECTORY METHOD

We use the decorrelation trajectory method, following the recent calculations for the influence of particle collisions on the diffusion in electrostatic turbulence [20]. The difference and the supplementary difficulty of the magnetic problem comes from structure (15) of the velocity $\tilde{\mathbf{v}}$ which is the product of two stochastic processes. They are statistically independent, but in the Lagrangian frame they are correlated through the trajectories due to the space dependence of the magnetic field fluctuations. The latter makes this problem strongly nonlinear. The trajectories also depend on the collisional velocity $\boldsymbol{\eta}_{\perp}$, and thus the velocity $\tilde{\mathbf{v}}$ is a triply stochastic process in the Lagrangian frame.

We now show that the problem is significantly simplified by first averaging all quantities over the perpendicular collisions.

We determine the collisional contributions to the perpendicular displacement:

$$\boldsymbol{\xi}(t)=\overline{\chi}_{\perp}^{1/2}\int_{0}^{t}\boldsymbol{\eta}_{\perp}(\tau)d\tau, \tag{16}$$

and make the change of variable $\mathbf{x}'(t)=\mathbf{x}(t)-\boldsymbol{\xi}(t)$ in Eq. (10), which introduces the collisional displacements in the argument of the magnetic field fluctuations:

$$\frac{d\mathbf{x}'}{dt}=M\,\tilde{\mathbf{b}}[\mathbf{x}'(t)+\boldsymbol{\xi}(t),z,t]\,\eta_{\parallel}(t). \tag{17}$$

We calculate the Eulerian correlation (EC) of the magnetic fluctuations $\tilde{\mathbf{b}}[\mathbf{x}+\boldsymbol{\xi}(t),z,t]$, averaged over both the magnetic field fluctuations and over the perpendicular collisional velocity. We calculate first the EC of the modified potential $\phi(\mathbf{x},z,t)\equiv\tilde{\phi}[\mathbf{x}+\boldsymbol{\xi}(t),z,t]$:

$$\begin{aligned}E&\equiv\langle\langle\tilde{\phi}[\mathbf{x}_1+\boldsymbol{\xi}(t_1),z_1,t_1]\tilde{\phi}[\mathbf{x}_2+\boldsymbol{\xi}(t_2),z_2,t_2]\rangle\rangle_{\perp}\\&=\langle A[\mathbf{x}_1+\boldsymbol{\xi}(t_1)-\mathbf{x}_2-\boldsymbol{\xi}(t_2),\,z_1-z_2,t_1-t_2]\rangle_{\perp}.\end{aligned} \tag{18}$$

The detailed calculation of E is given in Appendix A. The ''perpendicular'' average of the EC of the magnetic potential ϕ, $A(\mathbf{x},z,\tau)$, is transformed into $E(\mathbf{x},z,\tau)$ [Eq. (18)] that contains a supplementary time dependence in addition to that determined by the finite correlation time of the stochastic magnetic field

$$E(\mathbf{x},z,\tau)=\int d\boldsymbol{\xi}A(\mathbf{x}+\boldsymbol{\xi},z,\tau)\,P_{\perp}(\boldsymbol{\xi},\tau). \tag{19}$$

The Gaussian distribution function $P_{\perp}(\boldsymbol{\xi},\tau)$ is defined in Appendix A, Eq. (A3). As noted in Ref. [20], E is the solution of a diffusive equation and the effect of collisions consists in progressively smoothing out the EC of the magnetic potential and in eliminating asymptotically the $\mathbf{x}$ dependence of $E(\mathbf{x},z,\tau)$. Since the integral over $\mathbf{x}$ of E is constant, the time dependence introduced by collisions in Eq. (19) does not destroy the correlation but only spreads it out. Note that the average over the collisional parallel velocity was not performed at this stage: z in Eq. (18) is an Eulerian coordinate.

The problem of collisional particle motion in magnetic turbulence (10) and (11) is now formally reduced to a *doubly* (instead of triply) stochastic process; the former can be written in terms of the field $\mathbf{b}(\mathbf{x},z,t)$ generated by the modified potential $\phi(\mathbf{x},z,t)\equiv\tilde{\phi}[\mathbf{x}+\boldsymbol{\xi}(t),z,t]$:

$$\frac{d\mathbf{x}}{dt}=M\,\mathbf{b}(\mathbf{x},z,t)\,\eta_{\parallel}(t). \tag{20}$$

The Eulerian correlation of the components of $\mathbf{b}(\mathbf{x},z,t)$ are determined from the EC of potential (19) by equations similar to Eq. (6), with A replaced by E. The Langevin equation (20) is similar to the two-dimensional divergence-free problem studied in Ref. [19]. The velocity

$$\mathbf{v}(\mathbf{x},z,t)\equiv\mathbf{b}(\mathbf{x},z,t)\,\eta_{\parallel}(t) \tag{21}$$

has a much more complicated structure being determined by the product of two stochastic processes. However, the method developed in Ref. [19] can be used here: we will follow the same calculation steps as in Ref. [20].

First, we define a set of subensembles S of the realizations of the stochastic functions that have given values of the potential ϕ, of the magnetic field $\mathbf{b}$ and of the parallel velocity $\eta_{\parallel}$ in the point $\mathbf{x}=\mathbf{0}$, $z=0$ at time $t=0$:

$$\phi(\mathbf{0},0,0)=\phi^0,\quad \mathbf{b}(\mathbf{0},0,0)=\mathbf{b}^0,\quad \eta_{\parallel}(0)=\eta^0. \tag{22}$$

The correlation of the Lagrangian velocity (21) can be represented by a sum over the subensembles of the correlations appearing in each subensemble

$$L(t)=\int d\phi^0\mathbf{db}^0d\eta^0P(\mathbf{b}^0,\phi^0,\eta^0)\langle\mathbf{v}(\mathbf{0},0,0)\mathbf{v}[\mathbf{x}(t),z(t),t]\rangle_S \tag{23}$$

weighted by the probability $P(\mathbf{b}^0,\phi^0,\eta^0)$ of having $\mathbf{b}^0,\phi^0,\eta^0$ at $\mathbf{x}=\mathbf{0}$, $z=0$ and $t=0$, which is $P(\mathbf{b}^0,\phi^0,\eta^0)=P(b_1^0)P(b_2^0)P(\phi^0)P(\eta^0)$ with $P(X)=\exp(-X^2/2)/\sqrt{2\pi}$. This probability is a product of individual distributions, because the stochastic variables are not correlated in $\mathbf{x}=\mathbf{0}$, $z=0$, $t=0$. The point $\mathbf{x}=\mathbf{0}$, $z=0$ is taken as the initial condition for the trajectories determined from Eqs. (20) and (11). Since the initial velocity in the subensemble S is $\mathbf{v}(\mathbf{0},0,0)=\mathbf{b}^0\eta^0$ for all trajectories in S, the subensemble average in Eq. (23) is

$$\langle\mathbf{v}(\mathbf{0},0,0)\mathbf{v}(\mathbf{x}(t),z(t),t)\rangle_S=\mathbf{b}^0\eta^0\langle\mathbf{v}[\mathbf{x}(t),z(t),t]\rangle_S, \tag{24}$$

and thus the Lagrangian correlation $L(t)$ is determined by the average Lagrangian velocities in all subensembles. In order to evaluate these quantities, we need to calculate the average Eulerian velocity in the subensemble S,

$$\mathbf{V}^S(\mathbf{x},t)\equiv\langle\mathbf{v}[\mathbf{x},z(t),t]\rangle_S=\langle\mathbf{b}[\mathbf{x},z(t),t]\,\eta_{\parallel}(t)\rangle_S, \tag{25}$$

where $\langle\cdots\rangle_S$ is the average over the two stochastic processes restricted to the realizations in S and $z(t)$ is the stochastic parallel displacement obtained from Eq. (11). The details of the calculation of this quantity are given in Apendix B. The resulting average velocity (25) in the subensemble S is

$$\mathbf{V}^S(\mathbf{x},t)=\int dz\mathbf{B}^S(\mathbf{x},z,t)M_\parallel(z,t), \tag{26}$$

where

$$\mathbf{B}^S(\mathbf{x},z,t)\equiv\langle\mathbf{b}(\mathbf{x},z,t)\rangle_S=\left(\frac{\partial}{\partial x_2},-\frac{\partial}{\partial x_1}\right)\Phi^S(\mathbf{x},z,t), \tag{27}$$

with the average potential in the subensemble $\Phi^S(\mathbf{x},z,t)$ given by

$$\Phi^S(\mathbf{x},z,t)=\phi^0E(\mathbf{x},z,t)+b_i^0E_{i\phi}(\mathbf{x},z,t), \tag{28}$$

where $E_{i\phi}(\mathbf{x},z,t)=\langle b_i(\mathbf{0},0,0)\ \phi(\mathbf{x},z,t)\rangle=-\varepsilon_{ij}(\partial/\partial x_j)\times E(\mathbf{x},z,t)$. The second factor in Eq. (26) is

$$M_\parallel(z,t)=\left[\eta^0R(t)-\overline{\chi_\parallel}^{1/2}\mathcal{D}(t)[1-R(t)]\frac{\partial}{\partial z}\right]P^S(z,t), \tag{29}$$

where $P^S(z,t)$ is the probability of having a parallel displacement z at time t taken for the trajectories in the subensemble S: it is a Gaussian function defined in Appendix B, Eq. (B9), and the reduced parallel running diffusion coefficient is [see, Eq. (A5)]

$$\mathcal{D}(t)=\frac{1}{2}\frac{d\Psi(t)}{dt}=1-\exp(-t). \tag{30}$$

The next step in the decorrelation trajectory method consists of finding a *deterministic* trajectory $\mathbf{X}^S(t)$ in each subensemble S as the solution of the equation

$$\frac{d\mathbf{X}^S(t)}{dt}=M\mathbf{V}^S[\mathbf{X}^S(t),t] \tag{31}$$

with $\mathbf{X}^S(0)=\mathbf{0}$. Using Eqs. (26) and (27) one can show that this is a Hamiltonian system of equations which can be written as

$$\frac{dX^S(t)}{dt}=M\frac{\partial H^S(X^S,Y^S,t)}{\partial Y^S}\bigg|_{\mathbf{X}^S=\mathbf{X}^S(t)},$$
$$\frac{dY^S(t)}{dt}=-M\frac{\partial H^S(X^S,Y^S,t)}{\partial X^S}\bigg|_{\mathbf{X}^S=\mathbf{X}^S(t)}, \tag{32}$$

with the Hamiltonian

$$H^S(\mathbf{X}^S,t)=\int dz\ \Phi^S(\mathbf{X}^S,z,t)M_\parallel(z,t). \tag{33}$$

This Hamiltonian represents the average potential in the subensemble S. Its explicit expression is calculated for correlations (5) and (9). Since the stochastic magnetic field considered here is isotropic, the Hamiltonian could be simplified in a given subensemble by taking the x axis along $\mathbf{b}^0$. One obtains

$$H^S(X^S,Y^S,t)=b\,\eta^0(p+n_\perp(t)Y^S)n_\perp(t)\times\exp\left\{-\frac{1}{2}n_\perp(t)[(X^S)^2+(Y^S)^2]\right\}f_\parallel(t), \tag{34}$$

where

$$f_\parallel(t)=n_\parallel^{1/2}\{R-\overline{\chi_\parallel}n_\parallel(t)\mathcal{D}^2(t)[1-R(t)]\}\times\exp\left[-\frac{1}{2}(\eta^0)^2n_\parallel(t)\overline{\chi_\parallel}\mathcal{D}^2(t)\right], \tag{35}$$

$$n_\perp(t)\equiv[1+\overline{\chi_\perp}\Psi(t)]^{-1},\qquad n_\parallel(t)=[1+s^2(t)]^{-1}. \tag{36}$$

The parameters of the subensemble S are in Eq. (34), $b=|\mathbf{b}^0|$, $p\equiv\phi^0/b$, and η^0. The equations for the decorrelation trajectory (32) obtained from the Hamiltonian (34) are

$$\frac{dX^S}{dt}=Mb\,\eta^0n_\perp^2f_\parallel[1-pY^S-n_\perp(Y^S)^2]\times\exp\left(-\frac{1}{2}n_\perp[(X^S)^2+(Y^S)^2]\right), \tag{37}$$

$$\frac{dY^S}{dt}=Mb\,\eta^0n_\perp^2f_\parallel X^S(p+n_\perp Y^S)\times\exp\left(-\frac{1}{2}n_\perp[(X^S)^2+(Y^S)^2]\right). \tag{38}$$

The main assumption of the decorrelation trajectory method is the following (see, Ref. [20]): the average Lagrangian velocity is approximated by the average Eulerian velocity along the deterministic decorrelation trajectory

$$\langle\mathbf{v}[\mathbf{x}(t),z(t),t]\rangle_S\cong\mathbf{V}^S[\mathbf{X}^S(t),t], \tag{39}$$

where $\mathbf{X}^S(t)$ is the solution of Eqs. (37) and (38).

We finally obtain using Eqs. (23), (24), and (39) the autocorrelation of the perpendicular Lagrangian velocity $\mathbf{v}[\mathbf{x}(t),z(t),t]=\mathbf{b}[\mathbf{x}(t),z(t),t]\,\eta_\parallel(t)$ for arbitrary values of the four dimensionless parameters (12)–(14) and for given Eulerian correlations of the three stochastic processes that combine in equations of motion (3) and (4):

$$L(t;M,\overline{\chi_\parallel},\overline{\chi_\perp},\overline{\tau_c})=(\nu\lambda_\perp)^2M^2\frac{1}{2\pi}\int_0^\infty dp\times\int_0^\infty db\,b^3\exp\left[-\frac{b^2}{2}(p^2+1)\right]\times\int_{-\infty}^\infty d\eta^0\,\eta^0\exp\left(-\frac{\eta^{02}}{2}\right)V_1^S(\mathbf{X}^S(t),t). \tag{40}$$

The total perpendicular running diffusion coefficient is the sum of two terms: a direct contribution of the collisional velocity $\boldsymbol{\eta}_\perp$ obtained from Eq. (A4), and the contribution of the velocity (15),

$$D(t;M,\overline{\chi_\parallel},\overline{\chi_\perp},\overline{\tau_c})=\chi_\perp\mathcal{D}(t)+(\nu\lambda_\perp^2)D_{int}(t;M,\overline{\chi_\parallel},\overline{\chi_\perp},\overline{\tau_c}). \tag{41}$$

The latter is the time integral of the Lagrangian correlation (40) and can be written as

$$D_{int}(t;M,\overline{\chi_\parallel},\overline{\chi_\perp},\overline{\tau_c}) = \frac{M}{2\pi}\int_0^\infty dp\int_0^\infty db\, b^3\exp\left[-\frac{b^2}{2}(p^2+1)\right] \times\int_{-\infty}^{\infty}d\eta^0\,\eta^0\exp\left(-\frac{\eta^{02}}{2}\right)X^S(t), \tag{42}$$

where $X^S(t)$ is the component along the x axis of the solution of Eq. (32). It depends on the parameters M, $\overline{\chi_\parallel}$, $\overline{\chi_\perp}$, and $\overline{\tau_c}$ as well as on the shape of the Eulerian correlations. This contribution (42) results from the nonlinear interaction of the three stochastic processes. These results (40) and (41) are written as dimensional quantities. The asymptotic diffusion coefficient is

$$D(M,\overline{\chi_\parallel},\overline{\chi_\perp},\overline{\tau_c})=(\nu\lambda_\perp^2)[\overline{\chi_\perp}+D_{int}(M,\overline{\chi_\parallel},\overline{\chi_\perp},\overline{\tau_c})], \tag{43}$$

where $D_{int}(M,\overline{\chi_\parallel},\overline{\chi_\perp},\overline{\tau_c})$ is the limit for $t\to\infty$ of $D_{int}(t;M,\overline{\chi_\parallel},\overline{\chi_\perp},\overline{\tau_c})$.

A computer code that calculates the running diffusion coefficient starting from analytical expression (42) has been developed. It determines the decorrelation trajectories (32) for a large enough number of subensembles and performs the integrals in Eq. (42). The code was tested and the parameters in the numerical calculation were established using the analytical results concerning the subdiffusive transport. Namely, as shown in the following section, the asymptotic expression for the decorrelation trajectories and for the diffusion coefficient can be determined for an arbitrary M and $\overline{\chi_\parallel}$, in the case where $\overline{\chi_\perp}=0$ and $\overline{\tau_c}=\infty$. This provides a very good test for the code and permits the optimization of the choice of the parameters.

The analysis of the collisional particle transport in stochastic magnetic fields obtained by means of the decorrelation trajectory method results (40)–(43) is the subject of the subsequent three sections.

IV. SUBDIFFUSIVE TRANSPORT

We consider in this section a static stochastic magnetic field ($\tau_c\to\infty$) and the zero Larmor radius limit corresponding to negligible cross-field collisional diffusion, $\chi_\perp=0$. It is interesting to study separately this particular case, because it leads to a subdiffusive transport determined, as shown below, by two kinds of trapping processes. Moreover, the time dependence of the diffusion coefficient obtained for these particular conditions also allows the understanding of the scaling laws of the diffusion coefficient determined by the presence of a decorrelation mechanism.

First we study in this section the quasilinear case corresponding to small magnetic Kubo numbers, $K_m=\beta\lambda_\parallel/\lambda_\perp\ll 1$, where the magnetic lines are not trapped. In the limit $\lambda_\perp\to\infty$, an exact analytical solution has been determined in Ref. [16]. It was shown that particle perpendicular transport is subdiffusive with the running diffusion coefficient going asymptotically to zero as $t^{-1/2}$. This particular case is used here as a test for our general results (40)–(43). We show that the exact solution is found in this limit. Then the nonlinear problem corresponding to finite $\lambda_\perp$ and large magnetic Kubo number is studied. We show that the existence of magnetic line trapping does not change the asymptotic behavior of the diffusion coefficient: a similar subdiffusive regime is obtained from Eq. (42) with $D(t)\sim t^{-1/2}$. This nonlinear process has a strong effect but it is localized in time: it determines a transient decrease of $D(t)$. This transient effect is very important because it leads, as will be shown in the subsequent sections, to complex anomalous regimes when $\chi_\perp\neq 0$ or when $\overline{\tau_c}$ is finite.

In the limit $\lambda_\perp\to\infty$ the Lagrangian nonlinearity determined by the $\mathbf{x}$ dependence of the stochastic magnetic field disappears and the problem simplifies considerably. The equations for the decorrelation trajectories (38) reduce to

$$\frac{dX^S(t)}{dt}=-b\,\eta^0 f_\parallel(t),\qquad \frac{dY^S(t)}{dt}=0, \tag{44}$$

where the dimensional quantities were used. Thus the average Lagrangian velocity in S involved in the Lagrangian velocity correlation (40) is $V_1^S(t)=-b\,\eta^0 f_\parallel(t)$. The integrals over p b, and η^0 can easily be performed in Eq. (40), and one obtains

$$L_0(t;0,\overline{\chi_\parallel},0,\infty)=V^2\frac{1}{[1+\overline{\chi_\parallel}\Psi(t)]^{3/2}}\{R(t)[1+s^2(t)] -\overline{\chi_\parallel}\mathcal{D}^2(t)[1-R(t)]\}, \tag{45}$$

which after algebraic transformations becomes

$$L_0(t;0,\overline{\chi_\parallel},0,\infty)= V^2\frac{1}{[1+\overline{\chi_\parallel}\Psi(t)]^{1/2}}\left[R(t)-\frac{\overline{\chi_\parallel}\mathcal{D}^2(t)}{1+\overline{\chi_\parallel}\Psi(t)}\right]. \tag{46}$$

This is precisely identical with the exact analytical solution determined in Ref. [16] by means of a different method. The perpendicular running diffusion coefficient can be obtained by time integration of Eq. (46) as

$$D_0(t;0,\overline{\chi_\parallel},0,\infty)=(V^2/\nu)\frac{\mathcal{D}(t)}{[1+\overline{\chi_\parallel}\Psi(t)]^{1/2}}. \tag{47}$$

This exact solution obtained for $\lambda_\perp\to\infty$ is also valid for finite $\lambda_\perp$ as long as $M=K_m\overline{\chi_\parallel}^{1/2}\ll 1$. Actually this is the con-

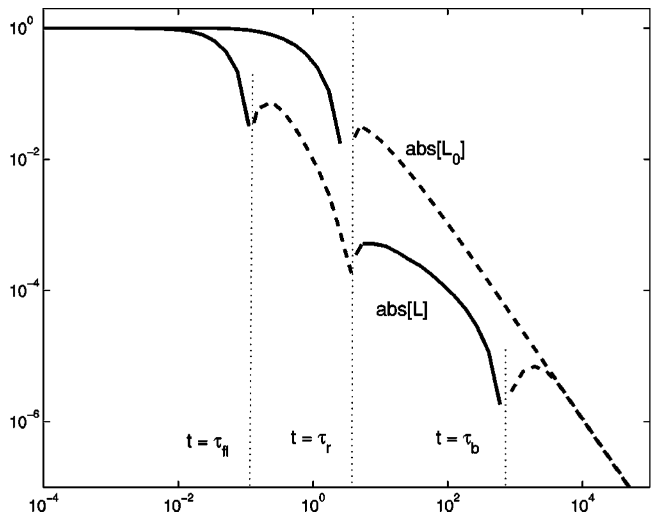

FIG. 1. The Lagrangian velocity correlation for the subdiffusive transport ($\overline{\chi_\perp}=0$, $\overline{\tau_c}=\infty$). $L_0(t)$ corresponds to $M\ll 1$ and is given by Eq. (46) and $L(t)$ is the nonlinear result obtained in the presence of magnetic line trapping at large K_m ($M=10$, $\overline{\chi_\parallel}=0.1$). The dashed parts of the two curves represent negative values of the Lagrangian correlations. The normalization constant is $V^2=(\lambda_\perp \nu M)^2$.

dition for neglecting the perpendicular displacements and the **x** dependence of the magnetic field fluctuations. Consequently, Eqs. (46) and (47) have physical relevance for tokamak plasmas, although $\lambda_\perp$ is of the order of 1 cm, and it is smaller than $\lambda_\parallel$ by at least a factor of 10^3. Due to the small values of β which are usually of the order of 10^{-4} the parameter M can be small.

The absolute value of $L_0(t;M,\overline{\chi_\parallel},0,\infty)$ and $D_0(t;M,\overline{\chi_\parallel},0,\infty)$ are plotted in Figs. 1 and 2. One can see that the Lagrangian correlation has a long negative tail at large t; its contribution exactly compensates the positive part appearing at small time such that its time integral is zero. More precisely, $D_0\sim t^{-1/2}$ for long time. The zero of the Lagrangian correlation (and the maximum of D_0) occurs at a time τ_r called the average return time; it is a decreasing function of $\overline{\chi_\parallel}$ scaling approximately as $\overline{\chi_\parallel}^{-1/2}$. It is remarkable to note that in the limiting case of the absence of collisions ($\nu=0$), Eq. (47) yields a finite diffusion coefficient [16]. In this case, $\chi_\parallel \nu = V_T^2/2$ (where V_T is the thermal velocity) and a small time expansion in Eq. (47) leads to the result of Jokipii and Parker [3], $D_{JP}=\beta^2\lambda_\parallel V_T/\sqrt{2}$. This is also well known as the Rechester and Rosenbluth collisionless diffusion coefficient [1], in the form $D_{RR}=D_m V_T$, where D_m is the diffusion coefficient of the magnetic lines (see also Refs. [10,12]).

Thus, the collisions determine a very strong change of the perpendicular transport, which is diffusive in the absence of collisions and becomes subdiffusive due to the parallel collisional motion. A physical interpretation of this subdiffusive behavior is presented in Ref. [15] in terms of a *parallel trajectory trapping process* determined by collisions which force the particles to return along the magnetic lines in the correlated zone, i.e., in the range of $\lambda_\parallel$ around the origin. Consequently, the Lagrangian velocities remain correlated. Since the parallel velocity changes its direction due to collisions, this long-time correlation is negative and thus determines the decay of the running diffusion coefficient $D_0(t)$.

We consider now the nonlinear case that corresponds to stochastic magnetic fields with finite $\lambda_\perp$ and large magnetic Kubo numbers $K_m>1$. A process of *magnetic line trapping* appears: the magnetic lines are constrained to turn around the small size contour surfaces of the potential $\phi(\mathbf{x},z)$ which are elongated along the z axis, making many turns before they can escape and possibly perform a long jump. The evolution along the z axis of the magnetic lines is a stochastic sequence of trapping events (helicoidal segments of the magnetic line) and long perpendicular jumps. This process is identical with the trapping of the trajectories determined by the $E\times B$ drift in a two-dimensional stochastic potential: the evolution of the magnetic lines along the z axis is described by the same equations as the time evolution of these trajectories. The process of trajectory trapping has been studied by means of the decorrelation trajectory method in Ref. [19], and the results obtained there can be applied to the stochastic magnetic lines.

The trapping of the magnetic lines has a strong influence on particle trajectories which follow the magnetic lines and evolve on helicoidal paths. We show that solutions (40)–(42) describe this trapping process: in the zero Larmor radius approximation ($\chi_\perp=0$) it leads to subdiffusive particle transport, provided that $\tilde{\mathbf{b}}$ is static ($\tau_c=\infty$). In this case, $n_\perp(t)=1$ and the Hamiltonian (34) depends on time only through the factor of $f_\parallel(t)$. It can be written as

$$H(X^S,Y^S,t)=f_\perp(X^S,Y^S)\, f_\parallel(t), \tag{48}$$

and consequently one can make a change of variable from t to $\tau(t)$ defined by

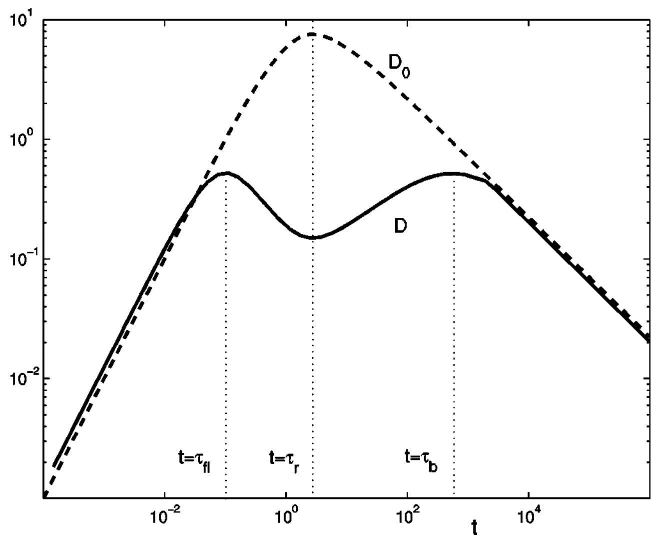

FIG. 2. The running diffusion coefficient corresponding to the Lagrangian velocity correlations in Fig. 1, $D_0(t)$ is the integral of $L_0(t)$ and is given by Eq. (47) and $D(t)$ is the integral of $L(t)$ and shows the effect of the magnetic line trapping. The normalization constant is $(\lambda_\perp^2 \nu)M$.

$$\frac{d\tau}{dt}=f_{\parallel}(t), \tag{49}$$

and the equations for the decorrelation trajectories become

$$\frac{dX^S}{d\tau}=M\frac{\partial f_{\perp}(X^S,Y^S)}{\partial Y^S},\quad \frac{dY^S}{d\tau}=-M\frac{\partial f_{\perp}(X^S,Y^S)}{\partial X^S}. \tag{50}$$

The function $\tau(t)$ has a maximum and then decays to zero. The solution of the time-independent Hamiltonian equations (50) is a periodic function of τ with $\mathbf{X}^S(\tau)$ lying on the closed path determined by $f_{\perp}(X^S,Y^S)=f_{\perp}(0,0)$. The size of the path depends only on p: it is infinite (straight line) at $p=0$ and decays to zero as p increases. The period is proportional to $(Mb\eta^0)^{-1}$. The decorrelation trajectories are thus obtained as $\mathbf{X}^S(\tau(t))$ where $\mathbf{X}^S(\tau)$ is the solution of Eq. (50). This shows that the trajectories wind around the closed paths (for an incomplete turn or for many turns, depending on M and on the parameters p, b, and η^0); at the time corresponding to the maximum of $\tau(t)$ they are all reflected and go back along the same path. Since $\tau(t)\to 0$ when $t\to\infty$, the asymptotic value of the decorrelation trajectories is $\mathbf{X}^S(\tau(t))\to\mathbf{X}^S(0)=\mathbf{0}$. All decorrelation trajectories eventually stop at the origin. This behavior of the decorrelation paths reflects the sticking of the particle trajectories on the trapped magnetic lines and the statistical characteristics of the collisional parallel motion. As $\mathbf{X}^S(\tau(t))\to\mathbf{0}$, the equation for diffusion coefficient (42) gives $D(t)\to 0$. Using Eqs. (49) and (35), the function $\tau(t)$ is shown to be $\tau(t)\cong(2\overline{\chi_{\parallel}}t)^{-1/2}$ at large t and with the solution of Eq. (50) at $\mathbf{X}^S\ll 1$ one obtains $X(t)\cong Mb\eta^0(2\overline{\chi_{\parallel}}t)^{-1/2}$. Upon substitution into Eq. (41) the running diffusion coefficient is obtained asymptotically as

$$D(t;M,\overline{\chi_{\parallel}},0,\infty)\to(\nu\lambda_{\perp}^2)M^2(2\overline{\chi_{\parallel}}t)^{-1/2}. \tag{51}$$

This subdiffusive behavior is identical with the asymptotic behavior obtained from quasilinear solution (47). Thus, the magnetic line trapping that appears for $K_m>1$ does not affect either the asymptotic time dependence of the running diffusion coefficient or its dependence on the parameters.

There is, however, a significant effect of the nonlinear process of magnetic line trapping, but it appears to be localized in time. It can be found by determining the whole time evolution of the running diffusion coefficient (42) using the computer code we have developed.

The results are presented in Figs. 1 and 2 compared to solution (46) and (47) obtained for $M\ll 1$. One can see that at small and large times the diffusion coefficient has the same expression as $D_0(t)$ in Eq. (47). For intermediate times a transient decrease of $D(t)$ appears. This is determined by the magnetic line trapping that is effective at times larger than the flight time over the perpendicular correlation length $\lambda_{\perp}$, which (in the unit considered here) is $\tau_{fl}=1/M$. As seen in Figs. 1 and 2, the running diffusion coefficient has a maximum at τ_{fl} and the Lagrangian velocity correlation becomes negative. Then the diffusion coefficient decreases due to the trapping of the magnetic lines. This process is represented by the decorrelation trajectories corresponding to subensembles with large values of the parameter $p=\phi^0/b$ which have performed many rotations around their paths (of small size) and their contribution cancels by mixing in the integrals in Eq. (41). Later in the evolution, another change of sign of the Lagrangian correlation is observed at $t=\tau_r$, the average return time for the parallel motion. At this moment $D(t)$ has a minimum while $D_0(t)$ has a maximum. It is determined by the parallel motion and more exactly by the collisions which force the particles to reverse their direction along the magnetic lines. This is reflected in the decorrelation trajectories, which all evolve back on their paths in the perpendicular plane at $t>\tau_r$. In the absence of magnetic line trapping (quasilinear conditions) this leads to the decay of the running diffusion coefficient, because the perpendicular displacement decreases in time and thus $D_0(t)$ decays at $t>\tau_r$. The effect is inverse in the presence of magnetic line trapping. The backward motion produces first the unmixing of the contribution of the trajectories that evolve on trapped magnetic lines. As time increases, the contributions of smaller and smaller decorrelation paths are recovered in the Lagrangian velocity correlation. The effect of magnetic line trapping that produced the decay of $D(t)$ in the interval (τ_{fl},τ_r) is washed out by the backward motion and $D(t)$ recovers its value at $t\sim\tau_{fl}$. At this moment τ_b, the correlation build-up time, $D(t)$ has a maximum. A positive bump appears in the Lagrangian velocity correlation due to the trajectories unwinding around the decorrelation paths. Finally, all decorrelation trajectories are "in phase" and approach the origin. This corresponds to the asymptotic regime in the evolution of the diffusion coefficient $D(t)$ which is the same with $D_0(t)$. Thus, the parallel collisional motion eliminates asymptotically the nonlinearity determined by the $\mathbf{x}$ dependence of the magnetic field fluctuations.

The above evolution of the diffusion appears whenever $\tau_{fl}<\tau_r$, and since $\tau_{fl}=M^{-1}$ and $\tau_r\approx\overline{\chi_{\parallel}}^{-1/2}$, the condition is $K_m>1$ which corresponds to the magnetic line trapping. When $\tau_{fl}>\tau_r$ (or $K_m\ll 1$), the running diffusion coefficient is given by Eq. (47).

We show in the subsequent sections that this rather nontrivial evolution of the running diffusion coefficient leads to anomalous diffusion regimes when a decorrelation mechanism is present.

V. DIFFUSIVE TRANSPORT INDUCED BY COLLISIONAL DECORRELATION

We analyze in this section the effect of the cross-field collisional diffusion ($\overline{\chi_{\perp}}\neq 0$) starting from general solution (40)–(42). The stochastic collisional velocity $\boldsymbol{\eta}_{\perp}(t)$ in Eq. (3) moves the particles away from the magnetic lines, and consequently it has a decorrelation effect leading to diffusive transport. This collisional motion determines a characteristic time, the perpendicular decorrelation time $\tau_{\perp}$. It is defined by the condition that the collisional diffusion covers the perpendicular correlation length, $2\chi_{\perp}\tau_{\perp}=\lambda_{\perp}^2$, and in the units chosen here it is $\overline{\tau_{\perp}}=(2\overline{\chi_{\perp}})^{-1}$. The stochastic magnetic field is considered here to be static ($\overline{\tau_c}=\infty$) for a better understanding of the collisional decorrelation.

As in the preceding section, a stochastic magnetic field with small Kubo number K_m that does not generate the magnetic line trapping is first considered. We show analytically that the already known results are reproduced by the decorrelation trajectory method. Then the nonlinear case is analyzed and new anomalous diffusion regimes are found. They are determined by the nonlinear interaction of the magnetic line trapping with the cross-field collisional diffusion.

In 1979 Kadomtsev and Pogutse [2] derived semiqualitatively an approximation for the cross-field diffusion coefficient. This approximation is essentially a weak-nonlinearity regime, in which the magnetic field fluctuations are nonchaotic. It will be shown that this diffusion coefficient is obtained from general equations (40)–(42), provided that $\tau_r < \tau_\perp < \tau_{fl}$. This condition is compatible with the relations found in Ref. [10] where a detailed study of the diffusion regimes in stochastic magnetic fields for fusion plasmas is presented using the Corrsin approximation. In this case the $\mathbf{X}^S$ dependence of the average velocity in Eqs. (37) and (38) can be neglected and the equations for the decorrelation trajectories are (44) corrected by a factor of $n_\perp^2(t)$ that multiplies the right hand side terms. This leads to the following form of the Lagrangian velocity correlation,

$$L_{KP}(t)=n_\perp^2(t)\,L_0(t), \tag{52}$$

where $L_0(t)$ is the subdiffusive Lagrangian velocity autocorrelation defined in Eq. (46). Because of the factor of $n_\perp^2(t)$, the integral of $L_{KP}(t)$ no longer vanishes, and yields a finite diffusion coefficient, D_{KP}. It can be estimated analytically by using a step approximation of the function $n_\perp(t)$,

$$n_\perp(t)\cong\begin{cases}1 & t<\tau_\perp\\ 0 & t>\tau_\perp .\end{cases} \tag{53}$$

It then follows that the diffusion coefficient is approximated as

$$D_{KP}\cong\int_0^{\tau_\perp} dt L_0(t)=-\int_{\tau_\perp}^{\infty} dt L_0(t), \tag{54}$$

because the integral of $L_0(t)$ from $t=0$ to infinity is zero. Using the very simple asymptotic form of $L_0(t)$ [obtained from Eq. (46) for $\tau>\tau_r$], the integral can be calculated analytically and one obtains (going to dimensional quantities)

$$D_{KP}\cong\beta^2\frac{\lambda_\parallel}{\lambda_\perp}\sqrt{\chi_\parallel\chi_\perp}, \tag{55}$$

which is the well-known Kadomtsev-Pogutse formula [2,10,12]. This result is thus reproduced by Eqs. (40)–(43).

We consider now the nonlinear case. When the time of flight τ_{fl} is smaller than the decorrelation time $\tau_\perp$, the space dependence of the magnetic field fluctuations cannot be neglected. It leads to magnetic line trapping. In the presence of a perpendicular collisional diffusivity the decorrelation trajectories obtained from Eq. (38) are no longer closed curves.

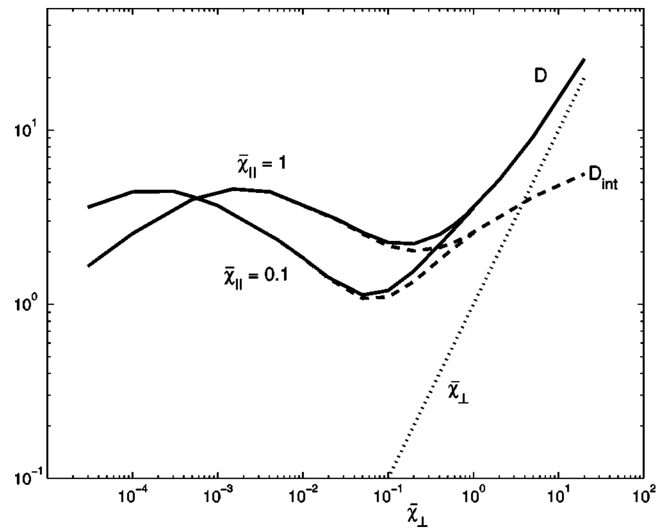

FIG. 3. The asymptotic diffusion coefficient as a function of $\overline{\chi_\perp}$. The total diffusion coefficient D (continuous lines) is compared with the direct collisional contribution $\overline{\chi_\perp}$ (dotted line) and with the interaction term D_{int} (dashed lines) for two values of $\overline{\chi_\parallel}$. The normalization constant is $\lambda_\perp^2\nu$, $M=10$, and $\overline{\tau_c}=\infty$.

However, trajectory winding can still be observed for some range of the parameters that define the subensembles. This means that the process of magnetic line trapping still exists. Compared to the decorrelation trajectories obtained with $\chi_\perp=0$, there now exist trajectories of several kinds. Some of them are still almost closed, performing a number of rotations and changing their sense of rotation at $t=\tau_r$. Some other trajectories stop before reaching τ_r. Most important, there exist open trajectories which actually have the most important contribution to the diffusion coefficient. This shows that both the perpendicular and the parallel trapping still exist. But due to the cross-field collisional diffusion, these two trapping processes appear only for a part of the decorrelation trajectories and are approximate or temporary. The perpendicular diffusion $\chi_\perp$ produces a releasing effect both for perpendicular and parallel components of particle motion. The asymptotic values of the decorrelation trajectories are not concentrated in the origin (as for $\chi_\perp=0$), but spread in the (X,Y) plane. Consequently, a finite value of the asymptotic diffusion coefficient is obtained from Eq. (41).

The asymptotic diffusion coefficient is determined from Eqs. (41)–(43) using the numerical code we have developed. Some results are presented in Fig. 3, where the asymptotic diffusion coefficient Eq. (43) is represented as a function of $\overline{\chi_\perp}$. The two components D_{int} and $\overline{\chi_\perp}$ are also represented. One can see that at small collisional diffusion $\overline{\chi_\perp}\ll 1$, the nonlinear interaction term largely dominates the collisional term while at large collisional diffusion $\overline{\chi_\perp}\gtrsim 1$, the nonlinear term is only a correction to $\overline{\chi_\perp}$. Thus, the subdiffusive transport appearing at $\overline{\chi_\perp}=0$ is transformed by a small collisional cross-field diffusion into a diffusive transport with a diffusion coefficient that can be several orders of magnitude larger than $\overline{\chi_\perp}$. The dependence of the diffusion coefficient on $\overline{\chi_\perp}$ is rather nontrivial. There is at very small $\overline{\chi_\perp}$, an increase of D up to a maximum which corresponds to $\tau_\perp \cong \tau_b$. Then, at larger $\overline{\chi_\perp}$, the nonlinear interaction of the parallel and perpendicular trapping with the collisional decorrelation generates an unusual transport regime, in which the effective diffusion coefficient decreases as the collisional diffusion $\overline{\chi_\perp}$ increases. A minimum of D is obtained when $\overline{\chi_\perp}$ determines a decorrelation time of the order of the return time of the parallel motion, $\tau_\perp \cong \tau_r$. At larger $\overline{\chi_\perp}$ (when $\tau_\perp < \tau_r$), the nonlinear contribution D_{int}, increases again with the increase of $\overline{\chi_\perp}$ but this contribution begins to be comparable and eventually negligible compared to the collisional diffusion coefficient $\overline{\chi_\perp}$.

We note that the above results obtained with the decorrelation trajectory method are not similar with the heuristic estimation of the asymptotic diffusion coefficient of Rechester and Rosenbluth [1]. This is possibly due to the fact that the trapping of the magnetic lines, which is implied in the above results, is neglected in the estimation [1] and also in the more detailed calculations presented in Ref. [10]. The latter estimation is based on the process of exponential increase of the average distance between two magnetic lines in a chaotic magnetic field, represented by the Kolmogorov length. An estimation of this length taking into account the trapping of the magnetic lines should be necessary in order to compare the results.

VI. DIFFUSIVE TRANSPORT IN TIME-DEPENDENT STOCHASTIC MAGNETIC FIELDS

In a time-dependent stochastic magnetic field with finite τ_c the configuration of the stochastic field $\tilde{\mathbf{b}}(\mathbf{x},z,t)$ changes, the magnetic lines move and consequently the perpendicular velocity of the particles is decorrelated leading to diffusive transport. We determine here the diffusion coefficient in such time-dependent fields in the limit of zero Larmor radius ($\overline{\chi_\perp}=0$), starting from general solution (40) and (41). The effect of time variation of the stochastic magnetic field on the effective diffusion was previously studied in Refs. [21–25], but only for weak magnetic turbulence ($K_m \ll 1$). We determine the effect of magnetic line trapping appearing in stochastic magnetic fields at $K_m > 1$.

The decorrelation trajectories obtained from Eqs. (37) and (38) are in this case (finite $\overline{\tau_c}$, $\overline{\chi_\perp}=0$) located on closed paths (except that for $p=0$). A typical trajectory rotates on the corresponding path, then it stops and turns back. Its velocity decays progressively and eventually the trajectory stops somewhere on its path. This is the modification determined by the time variation of the magnetic field: all decorrelation trajectories stop at a time of the order of $\overline{\tau_c}$. As follows from Eq. (42), the running diffusion coefficient saturates. Depending on the relation between the decorrelation time $\overline{\tau_c}$ and the three characteristic times of this motion, τ_{fl}, τ_r, τ_b (see Fig. 2) several diffusion regimes are obtained. In time-dependent magnetic fields, at $t<\tau_c$, the running diffusion coefficient is approximately the same as that obtained for $\overline{\tau_c}\to\infty$, and later, at $t>\tau_c$, $D(t)$ saturates. Thus, the asymptotic diffusion coefficient can be evaluated as

$$\lim_{t\to\infty} D(t;M,\overline{\chi_\parallel},0,\overline{\tau_c}) \cong D(\overline{\tau_c};M,\overline{\chi_\parallel},0,\infty), \tag{56}$$

using the running diffusion coefficient obtained in the static case. Hence, it can be approximated by the value of the running diffusion coefficient for the subdiffusive case at $t=\overline{\tau_c}$. Some results are presented in Fig. 4 where the asymptotic diffusion coefficient obtained from Eq. (42) for finite $\overline{\tau_c}$ is compared to the subdiffusive running diffusion coefficient represented in Fig. 2. One can see that approximation (56) is rather good for all values of $\overline{\tau_c}$.

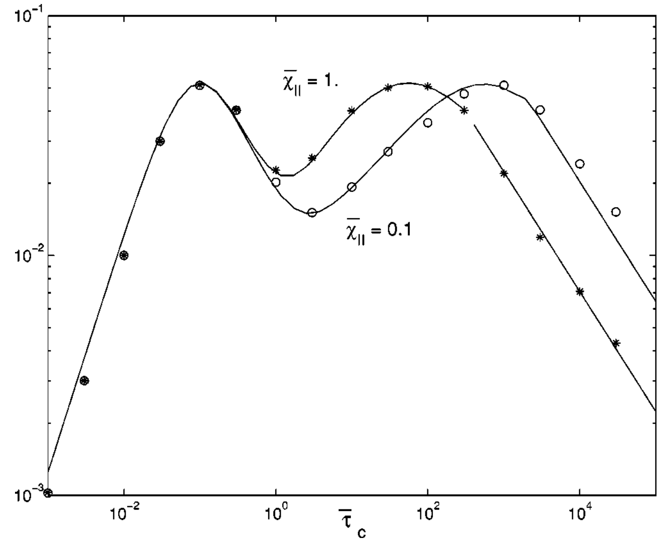

FIG. 4. The asymptotic diffusion coefficient as a function of $\overline{\tau_c}$ for $\overline{\chi_\parallel}=0.1$ (circles) and $\overline{\chi_\parallel}=1$ (stars). The continuous lines represent the running diffusion coefficient as a function of t for the subdiffusive transport corresponding to static magnetic fields ($\overline{\tau_c}=\infty$). The normalization constant is $(\lambda_\perp^2 \nu)M^2$, $M=10$, $\overline{\chi_\perp}=0$.

The following diffusion regimes can be observed in Fig. 4, in the nonlinear conditions when the trapping of the magnetic lines is effective ($\tau_{fl}<\tau_r$, or $K_m>1$). The quasilinear regime at small correlation times ($\overline{\tau_c}<\tau_{fl}$) with $D_0 \approx M^2\overline{\tau_c}$ is characterized by a fast time variation which prevents the trajectory trapping. At larger correlation times ($\tau_{fl}<\overline{\tau_c}<\tau_r$) the magnetic lines can be trapped before the stochastic magnetic field changes and the parallel motion is ballistic. In these conditions the diffusion regime is similar to that described in Ref. [20] for the electrostatic turbulence: the diffusion coefficient decreases with the increase of $\overline{\tau_c}$. A minimum of the diffusion coefficient appears at $\overline{\tau_c}\cong\tau_r$, followed by an anomalous increase determined by the interaction of the parallel trapping with the magnetic line trapping which generates correlation of the Lagrangian velocities. At very large correlation times ($\overline{\tau_c}>\tau_b$) the diffusion coefficient decreases as $D\approx K_m^2\overline{\tau_c}^{-1/2}\overline{\chi_\parallel}^{1/2}$.

We note that the regimes obtained for $\overline{\tau_c}<\tau_{fl}$ and for $\overline{\tau_c}>\tau_b$ are similar to those reported in Refs. [21,22,6]. But instead of the plateau found there at intermediate $\overline{\tau_c}$, we obtain here a smaller diffusion coefficient with a more complicated behavior. This is the effect of stochastic magnetic line trapping: it leads to the decrease of the effective diffusion coefficient with the increase of $\overline{\tau_c}$ when the parallel motion is ballistic and, on the contrary, to the increase of D with the increase of $\overline{\tau_c}$ when the parallel motion is diffusive. A minimum of the diffusion coefficient appears at the resonance condition $\overline{\tau_c} \cong \tau_r$. As seen in Fig. 4, this nonlinear effect can significantly reduce the diffusion.

VII. CONCLUSIONS

We have studied here the transport of collisional particles in stochastic magnetic fields using the decorrelation trajectory method. We have derived analytical expressions for the running diffusion coefficient and for the Lagrangian velocity correlation in terms of a set of deterministic trajectories. They are defined in subensembles of the realizations of the stochastic field as a solution of differential (Hamiltonian) equations that depend on the given shape of the Eulerian correlation of the stochastic potential. They are approximations of the subensemble average trajectories, and represent the dynamics of the decorrelation of the Lagrangian velocity. Since, in general, the equations for the decorrelation trajectories cannot be solved analytically, a computer code was developed for this purpose and for determining the running diffusion coefficient for arbitrary values of the four parameters of this problem and for given Eulerian correlation of the potential.

We have shown that this rather complicated triple stochastic process is characterized by two kinds of trajectory trappings and contains two decorrelation mechanisms. The latter are produced by the collisional cross-field diffusion $\overline{\chi_\perp}$ and/or by the time variation of the stochastic magnetic field.

One of the trapping processes concerns the parallel motion and is determined by collisions which constrain the particles to return in the already visited places with probability one. This parallel trapping leads to a subdiffusive transport in the absence of a decorrelation mechanism. This already known process is recovered by our method. The second kind of trapping concerns the magnetic lines that wind around the extrema of the potential at $K_m > 1$. The effects of the magnetic line trapping on the collisional particle transport is studied. We show that in the absence of a decorrelation mechanism, the magnetic line trapping determines a transient decay of the running diffusion coefficient $D(t)$ appearing at t in the interval (τ_{fl}, τ_r), i.e., before the parallel trapping is effective. The simultaneous action of both trapping processes determine a nonlinear build-up of Lagrangian velocity correlation, and eventually the parallel motion washes out the effect of the magnetic line trapping. Consequently, the asymptotic behavior of the running diffusion coefficient is exactly the same as in the quasilinear conditions when the stochastic magnetic field does not generate any magnetic line trapping.

The effect of the two decorrelation mechanisms is studied afterwards. We show that the effective diffusion coefficient and its dependence on the parameters result from a competition between the trapping and the decorrelation processes, and more precisely from the temporal ordering of the characteristic times of these processes. Each one of the two decorrelation mechanisms leads to the already known diffusion laws when the magnetic line trapping is not present ($K_m \ll 1$). The trapping of the magnetic lines (at $K_m > 1$) produces a complicated nonlinear interaction between the three stochastic processes which determines new scaling laws of the diffusion coefficient. They appear when the decorrelation time is longer than the flight time τ_{fl}, but smaller than the correlation build-up time τ_b. The first condition ensures the magnetic line trapping and the second prevents the elimination of this trapping effect by the parallel collisional motion. A particularly interesting regime is obtained for collisional decorrelation and consists of an effective diffusion coefficient that decreases when the collisional perpendicular diffusion increases (Fig. 3).

This rather complex dependence of the diffusion coefficients on the plasma parameters can be used in experiments for controlling the transport. Even without changing the characteristics of the stochastic magnetic field, the cross-field diffusion coefficient can be strongly influenced by the parameters that describe particle collisions. A minimum of the diffusion coefficient was obtained for decorrelation times of the order of the average return time for the parallel motion.

Several directions of research can be envisaged for the further development of the present work. One consists in studying the effect of trajectory fluctuations in the subensemble. Another important extension should be the estimation of the particle density distribution as was already done for simpler cases [14,26]. We also expect interesting effects from the periodic configuration of the magnetic field which determines magnetic islands and resonant surfaces.

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APPENDIX A

The average of the potential autocorrelation over the perpendicular collisional velocity, Eq. (18), can be calculated using the two-point Gaussian probability density,

$$E = \int \int \mathbf{d}\xi_1 \mathbf{d}\xi_2 A[\mathbf{x}_1 - \mathbf{x}_2 + \boldsymbol{\xi}_1 - \boldsymbol{\xi}_2, z_1 - z_2, t_1 - t_2] \times P_2(\boldsymbol{\xi}_1, t_1; \boldsymbol{\xi}_2, t_2), \tag{A1}$$

where $P_2(\boldsymbol{\xi}_1, t_1; \boldsymbol{\xi}_2, t_2)$ is the probability density for having $\boldsymbol{\xi}(t_1) = \boldsymbol{\xi}_1$ and $\boldsymbol{\xi}(t_2) = \boldsymbol{\xi}_2$. It is determined as the average over perpendicular collisions of the corresponding product of δ functions:

$$P_2(\boldsymbol{\xi}_1, t_1; \boldsymbol{\xi}_2, t_2) = \langle \delta(\boldsymbol{\xi}(t_1) - \boldsymbol{\xi}_1) \delta(\boldsymbol{\xi}(t_2) - \boldsymbol{\xi}_2) \rangle_\perp .$$

This probability can be calculated using the Fourier representation of the δ functions and the cumulant expansion of the resulting exponential. Since the collisional displacements are Gaussian, only the first two cumulants appear. One obtains the two-point probability density for the perpendicular collisional displacements as

$$P_2=\int\int d\mathbf{q}_1 d\mathbf{q}_2 \exp\left(i\mathbf{q}_1\cdot\boldsymbol{\xi}_1+i\mathbf{q}_2\cdot\boldsymbol{\xi}_2-\frac{q_1^2\langle\xi^2(t_1)\rangle_c}{2} -\frac{q_2^2\langle\xi^2(t_2)\rangle_c}{2}-\langle\mathbf{q}_1\cdot\boldsymbol{\xi}(t_1)\,\mathbf{q}_2\cdot\boldsymbol{\xi}(t_2)\rangle_c\right) \tag{A2}$$

which, introduced in Eq. (A1), using the dependence of the EC on the difference $\boldsymbol{\xi}=\xi_1-\boldsymbol{\xi}_2$ and performing the integrals over $\mathbf{q}_1$, $\mathbf{q}_2$, and $\boldsymbol{\xi}_1$, yields Eq. (19) of the main text. The one-point probability density for the perpendicular collisional displacements $P_\perp(\boldsymbol{\xi},\tau)$ is

$$P_\perp(\boldsymbol{\xi},\tau)=\frac{1}{2\pi\langle\xi^2(\tau)\rangle_c}\exp\left(-\frac{\xi^2}{2\langle\xi^2(\tau)\rangle_c}\right). \tag{A3}$$

The mean square displacement for the collisional perpendicular displacements is

$$\langle[\boldsymbol{\xi}(t_2)-\boldsymbol{\xi}(t_1)]^2\rangle_c=\langle\xi^2(\tau)\rangle_c=\overline{\chi_\perp}\Psi(\tau), \tag{A4}$$

where $\tau\equiv|t_2-t_1|$ and $\Psi(\tau)$, the reduced mean square perpendicular collisional displacement, is

$$\Psi(\tau)=2\int_0^\tau(\tau-t)R(t)dt=2[\tau+\exp(-\tau)-1]. \tag{A5}$$

APPENDIX B

In order to calculate the average of the Eulerian velocity in the subensemble S, Eq. (25), we first note that the stochastic parallel displacement $z(t)$ is obtained from Eq. (11) as

$$z(t)=\overline{\chi_\parallel}^{1/2}\int_0^t d\tau\,\eta_\parallel(\tau). \tag{B1}$$

Next, we note that the ith component of the average velocity in S is defined by the following conditional average:

$$V_i^S=\frac{\langle\langle b_i[\mathbf{x},z(t),t]\,\eta_\parallel(t)\ \delta[\mathbf{b}^0-\mathbf{b}(\mathbf{0},0,0)]\,\delta[\phi^0-\phi(\mathbf{0},0,0)]\,\delta[\eta^0-\eta_\parallel(0)]\rangle\rangle_\parallel}{P(\mathbf{b}^0,\phi^0,\eta^0)}. \tag{B2}$$

Introducing a function $\delta[z-z(t)]$ and using the statistical independence of $\mathbf{b}$ and $\eta_\parallel$ one can write

$$V_i^S=\int dz\,\frac{\langle b_i(\mathbf{x},z,t)\ \delta[\mathbf{b}^0-\mathbf{b}(\mathbf{0},0,0)]\ \delta[\phi^0-\phi(\mathbf{0},0,0)]\rangle}{P(\mathbf{b}^0,\phi^0)}\times\frac{\langle\eta_\parallel(t)\ \delta[z-z(t)]\,\delta[\eta^0-\eta_\parallel(0)]\rangle_\parallel}{P(\eta^0)}. \tag{B3}$$

The first average over the stochastic magnetic field represents the subensemble average of $b_i(\mathbf{x},z,t)$ in S and is given by Eq. (27) of the main text. The average potential in the subensemble S, $\Phi^S(\mathbf{x},z,t)\equiv\langle\phi(\mathbf{x},z,t)\rangle_S$, is calculated as in Ref. [20] and is obtained in the form of Eq. (28) of the main text.

The second average in Eq. (B3) over the collisional parallel velocity can be written using the Fourier representation of the δ functions as

$$M_\parallel\equiv\langle\eta_\parallel(t)\,\delta[z-z(t)]\,\delta[\eta^0-\eta_\parallel(0)]\rangle_\parallel\frac{1}{P(\eta^0)} =\frac{1}{P(\eta^0)}\int\int dk\,dq\,\exp(-ikz-iq\eta^0)\times\langle\eta_\parallel(t)\exp[ikz(t)+iq\eta_\parallel(0)]\rangle_\parallel. \tag{B4}$$

The average in this equation can be calculated as the derivative with respect to a of the following average, evaluated in $a=0$,

$$\langle\exp[a\eta_\parallel(t)+ikz(t)+iq\eta_\parallel(0)]\rangle_\parallel =\exp\left[-\frac{a^2}{2}-\frac{k^2}{2}\langle z^2(t)\rangle_c-\frac{q^2}{2}+iak\langle\eta_\parallel(t)z(t)\rangle_c +iaqR(t)-kq\langle\eta_\parallel(0)z(t)\rangle_\parallel\right], \tag{B5}$$

where

$$\langle\eta_\parallel(0)z(t)\rangle_\parallel=\overline{\chi_\parallel}^{1/2}\int_0^t d\tau R(\tau)=\overline{\chi_\parallel}^{1/2}\mathcal{D}(t), \tag{B6}$$

$$\langle\eta_\parallel(t)z(t)\rangle_\parallel=\overline{\chi_\parallel}^{1/2}\int_0^t d\tau R(t-\tau)=\overline{\chi_\parallel}^{1/2}\mathcal{D}(t), \tag{B7}$$

$$\langle z^2(t)\rangle_\parallel=\int_0^t\int_0^t d\tau_1 d\tau_2 R(|\tau_1-\tau_2|)=\overline{\chi_\parallel}\Psi(t). \tag{B8}$$

For the correlation R in Eq. (9), the reduced parallel running diffusion coefficient is defined in Eq. (30), and the reduced mean square parallel displacement is $\Psi(t)$ defined in Eq. (A5), the same as for the perpendicular collisional displacement.

Straightforward calculations lead from Eq. (B4) to the parallel average $M_\parallel$, given in the main text in Eq. (29), in which $P^S(z,t)$ is the probability of having a parallel displacement z at time t taken for the trajectories in the subensemble S. This was obtained as a Gaussian distribution with an average displacement $\langle z(t)\rangle_S$ and a modified dispersion $s(t)=\langle[z(t)-\langle z(t)\rangle_S]^2\rangle_S^{1/2}$,

$$P^S(z,t)=\frac{1}{\sqrt{2\pi}s(t)}\exp\left[-\frac{[z-\langle z(t)\rangle_S]^2}{2s^2(t)}\right]. \tag{B9}$$

The parallel average displacement is the integral of the parallel average velocity in S,

$$\langle \eta_\parallel(t)\rangle_S=\eta^0 R(t) \tag{B10}$$

and is obtained as

$$\langle z(t)\rangle_S=\eta^0\overline{\chi_\parallel}^{1/2}\mathcal{D}(t). \tag{B11}$$

The parallel dispersion of the trajectories in S is

$$s^2(t)=\langle z^2(t)\rangle-\overline{\chi_\parallel}\mathcal{D}^2(t)=\overline{\chi_\parallel}(\Psi(\tau)-\mathcal{D}^2(t)). \tag{B12}$$

Thus the dispersion of the parallel component of the trajectories in a subensemble S is always smaller than the dispersion of the whole set of trajectories $\langle z^2(t)\rangle$. It grows slowly (as t^3) at small t and at $t\gg1$, it reaches the global dispersion $\langle z^2(t)\rangle$. The parallel running diffusion coefficient in the subensemble S is $\mathcal{D}_\parallel^S(t)=\overline{\chi_\parallel}\mathcal{D}(t)[1-R(t)]$. It behaves at small time as t^2 and at $t\gg1$, it is equal to the global diffusion coefficient of the whole set of trajectories, $\overline{\chi_\parallel}\mathcal{D}(t)$.

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